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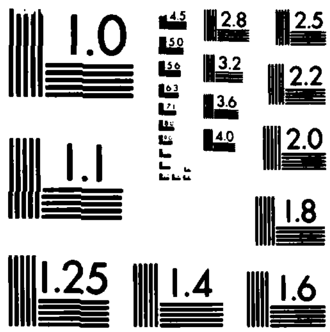
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ASYMPTOTIC FIELDS OF A PERFECTLY-PLASTIC, PLANE-STRESS MODE II GROWING CRACK

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Abstract

The asymptotic near-tip stress and velocity fields are presented for a plane-stress mode II crack propagating quasi-statically in an elastic-perfectly plastic Mises solid. The solution is found to have fully continuous stress and velocity fields, and a configuration similar to that of the anti-plane strain problem: a singular centered fan plastic sector ahead of the crack, followed by an elastic unloading sector and a constant stress plastic sector extending to the crack flank. The impossibility of a plane-stress mode I crack solution having these properties is also discussed.

1. Introduction

Rice (1982) presents a complete analysis of the asymptotic structure of the near-tip stress and deformation fields of a crack growing quasi-statically in an elastic-perfectly plastic solid. There, all possible solutions to the governing equations are given for anti-plane strain, plane strain and plane stress.

In anti-plane strain, Chitaley and McClintock (1971) gave the first successful assembly of sectors for the Mises material. In plane strain, Slepyan (1974) gave the corresponding assembly of sectors for the Tresca material in both modes I and II. Independently, Gao (1980) and Rice *et al.* (1980) produced results for the Mises material in

mode I ($\nu = 1/2$), and Drugan *et al.* (1982) generalised these results to the case of $\nu \neq 1/2$. In plane stress, the mode I problem has remained most elusive; in this note we present a solution to the mode II problem, and throw some light on the mode I problem.

2. Formulation

With reference to Figure 1, let x_i (i = 1,2,3) be a Cartesian coordinate system of fixed orientation travelling with the crack tip such that the x_3-axis coincides with the straight crack front. Also let $\mathbf{e}_i$ be the unit vector corresponding to the x_i direction. Similarly, let r, θ be polar coordinates corresponding to x_α ($\alpha = 1,2$) and $\mathbf{e}_r$, $\mathbf{e}_\theta$ be the corresponding unit vectors. The crack tip moves with velocity $\mathbf{V} = V\mathbf{e}_1$ with respect to the stationary coordinate system X_i. In this asymptotic analysis the material derivative is given by

$$(\ )^{\cdot} = -V(\ )_{,1} \tag{1}$$

The dependent variables of this problem are the in-plane components of the stress tensor $\boldsymbol{\sigma}$, and the velocity vector $\mathbf{v}$ (v_3 does not enter the formulation). The governing equations are equilibrium

$$\nabla\cdot\boldsymbol{\sigma} = \mathbf{0} \tag{2}$$

and the constitutive relations

$$\mathbf{D} = (1/E)\,(1+\nu)\,\boldsymbol{\Sigma} - (\nu/E)\,\mathrm{Tr}(\boldsymbol{\Sigma})\,\mathbf{I} + \Lambda^{\cdot}\,\mathbf{S} \tag{3}$$

where: E is the modulus of elasticity; $\mathbf{I} = \mathbf{e}_i\mathbf{e}_i$ is the identity tensor; $\mathbf{S} = \boldsymbol{\sigma} - (1/3)\,\mathrm{Tr}(\boldsymbol{\sigma})\,\mathbf{I}$ is the stress-deviator tensor; $\mathbf{D} = (1/2)[\nabla\mathbf{v} + (\nabla\mathbf{v})^t]$ is the strain-rate tensor; $\boldsymbol{\Sigma} = \boldsymbol{\sigma}^{\cdot}$ is the stress-rate tensor; and $\Lambda^{\cdot}$ is a scalar such that (i) $\Lambda^{\cdot} = 0$ for elastic unloading, (ii) $\Lambda^{\cdot} \geq 0$ for plastic loading, when the equations are supplemented by the Mises yield condition

$$\sigma_e = [(3/2)\mathbf{S}:\mathbf{S}]^{1/2} = \sqrt{3}\,\tau_o \tag{4}$$

with τ_o denoting the yield stress in shear.

The boundary conditions of this problem are

$$\sigma_{rr}(r,0) = \sigma_{\theta\theta}(r,0) = v_r(r,0) = 0 \tag{5}$$

as required by mode II symmetry, and

$$\sigma_{r\theta}(r,\pi) = \sigma_{\theta\theta}(r,\pi) = 0 \tag{6}$$

because the crack faces are traction-free.

Rice (1982) has shown that the governing equations admit only three types of asymptotic solutions. Thus near the crack tip we can only have three types of sectors: elastic sectors, and plastic sectors of either the constant stress or centered fan type. Here we will look for a solution with a centered fan sector ahead of the crack ($0 < \theta < \theta_1$), followed by an elastic sector ($\theta_1 < \theta < \theta_2$) and a constant stress sector extending to the crack face ($\theta_2 < \theta < \pi$).

According to Drugan and Rice (1984) we need to impose continuity of all the components of the stress tensor across each elastic-plastic boundary

$$[\sigma_{r\theta}] = [\sigma_{rr}] = [\sigma_{\theta\theta}] = 0 \tag{7}$$

where [ ] denotes the jump in a quantity as θ increases infinitesimally. They also show that we can impose continuity of the velocity vector, unless the stress state at the boundary meets certain specific conditions, in which case discontinuities in the [redacted] components of the velocity cannot be ruled out. Here we will look for a solution with a continuous velocity so that we impose

$$[v_r] = [v_\theta] = 0 \tag{8}$$

3. Solution

The leading order terms in the asymptotic expansion of the stress and deformation fields in the three sectors can easily be calculated, and are given below. Note that the boundary conditions have already been imposed in these expressions.

(i) *Centered fan sector*

$$\sigma_{r\theta} = \tau_o \cos\theta \qquad \sigma_{rr} = -\tau_o \sin\theta \qquad \sigma_{\theta\theta} = -2\tau_o \sin\theta \tag{9}$$

$$\begin{aligned} v_r &= -(3/2)V(\tau_o/E)\sin 2\theta \ln(r/R) \\ v_\theta &= 3V(\tau_o/E)\,[1-(4/5)\cos^2\theta + B(\cos\theta)^{-1/2}]\ln(r/R) \end{aligned} \tag{10}$$

$$\Lambda^{\cdot} = -(3/2)(V/E)\,[\,(6/5)\cos\theta + B(\cos\theta)^{-1/2}]\ln(r/R)/r \tag{11}$$

(ii) *Elastic sector*

$$\begin{aligned} \sigma_{12} &= (\tau_o/4)\,[\,A_1(2\theta + \sin 2\theta) - A_2\cos 2\theta + C_{12}] \\ \sigma_{11} &= (\tau_o/4)\,[\,4A_1 \ln|\sin\theta| + A_1\cos 2\theta + A_2(2\theta + \sin 2\theta) + C_{11}] \\ \sigma_{22} &= (\tau_o/4)\,[\,-A_1\cos 2\theta + A_2(2\theta - \sin 2\theta) + C_{22}] \end{aligned} \tag{12}$$

$$\begin{aligned} v_1 &= V(\tau_o/E)\,A_1 \ln(r/R) \\ v_2 &= V(\tau_o/E)\,A_2 \ln(r/R) \end{aligned} \tag{13}$$

(iii) *Constant stress sector*

$$\sigma_{12} = 0 \qquad \sigma_{11} = \sqrt{3}\,\tau_o \qquad \sigma_{22} = 0 \tag{14}$$

$$\begin{aligned} v_1 &= V(\tau_o/E)\,D_1 \ln(r/R) \\ v_2 &= V(\tau_o/E)\,D_2 \ln(r/R) \end{aligned} \tag{15}$$

$$\Lambda^{\cdot} = (1/\sqrt{3})(V/E)(1/\cos\theta)(D_1 + D_2\tan\theta)/r \tag{16}$$

These fields involve ten unknown constants (A_1, A_2, B, C_{12}, C_{11}, C_{22}, D_1, D_2, θ_1 and θ_2), and must be subjected to the five continuity conditions given by (7) and (8) across the two boundaries for a total of ten conditions. A solution to this nonlinear algebraic system was found with

$$\begin{aligned} &\theta_1 \approx 13.31383^\circ && \theta_2 \approx 179.61254^\circ \\ &A_1 = D_1 \approx -0.68994 && A_2 = D_2 \approx -0.00387 \\ &B \approx -0.18814 && C_{12} \approx 0.26953 \\ &C_{11} \approx -0.38413 && C_{22} \approx -0.04160 \end{aligned} \tag{17}$$

and the associated stress and velocity fields are depicted in Figures 2 and 3. Note that the yield condition is nowhere violated, and in particular that $\sigma_e < 1$ for $\theta_1 < \theta < \theta_2$. Also note that $\Lambda^{\cdot} > 0$ near the tip in both plastic sectors.

4. Concluding remarks

The results of this problem agree in form with those of the other two anti-symmetric cases, anti-plane strain mode III and plane-strain mode II. Thus it is found in all these cases that the solution has continuous stress and velocity fields, and the same configuration: a plastic sector ahead of the crack which produces singular strains, followed by an elastic unloading sector and a reverse plastic flow sector on the crack flank which does not produce any further singular straining.

Vis-à-vis the plane-stress mode I problem, we find that assuming a similar assembly of sectors does not yield a solution with continuous stresses and velocities. To see this, we remark that the velocities in the centered fan sector would be (Rice, 1982)

$$\begin{aligned} v_r &= -3V(\tau_o/E)\sin^2\theta \ln(r/R) \\ v_\theta &= -3V(\tau_o/E)(\sin\theta)^{-1/2}\left[\int_{\bullet}^{\bullet}(\sin\phi)^{1/2}\cos 2\phi \, d\phi + B\right]\ln(r/R) \end{aligned} \tag{18}$$

Mode I symmetry would then require

$$v_\theta(r,0) = 0 \tag{19}$$

This would make B vanish, which in the context of the previous formulation would leave only nine unknowns to satisfy ten conditions. Hence, the impossibility of finding a continuous solution to the plane-stress mode I problem with the given configuration of sectors.

A somewhat similar situation appeared in the plane-strain mode I problem where a discontinuity in the tangential component of the velocity, consistent with the material model, had to be admitted. Thus it is conceivable that discontinuities in the velocity may

have to be introduced in the solution of the plane-stress mode I problem. Pan (1984) has considered such discontinuities.

Finally, we point out that in addition to the plane-stress mode I problem, the Mises plane-strain mode II problem with $\nu \neq 1/2$ remains to be solved. For $\nu = 1/2$ the solution obtained by Slepyan (1974) for the Tresca material also holds for the Mises material.

Acknowledgement

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Figures

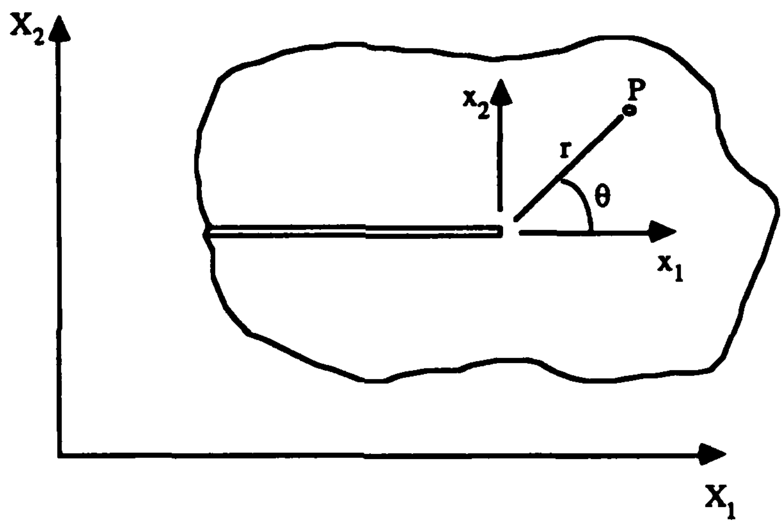

Figure 1. Crack-tip geometry.

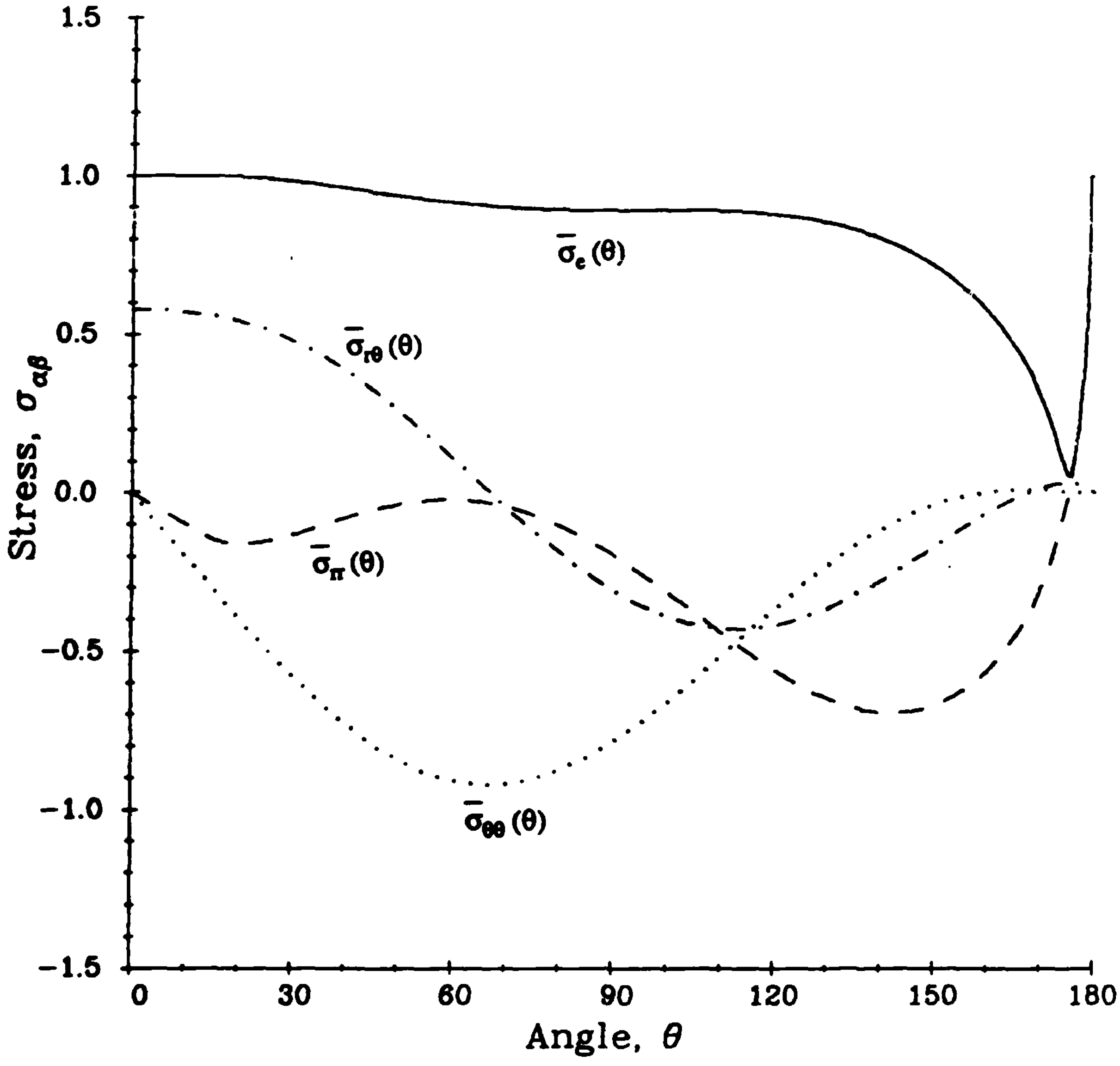

Figure 2. Stress distribution

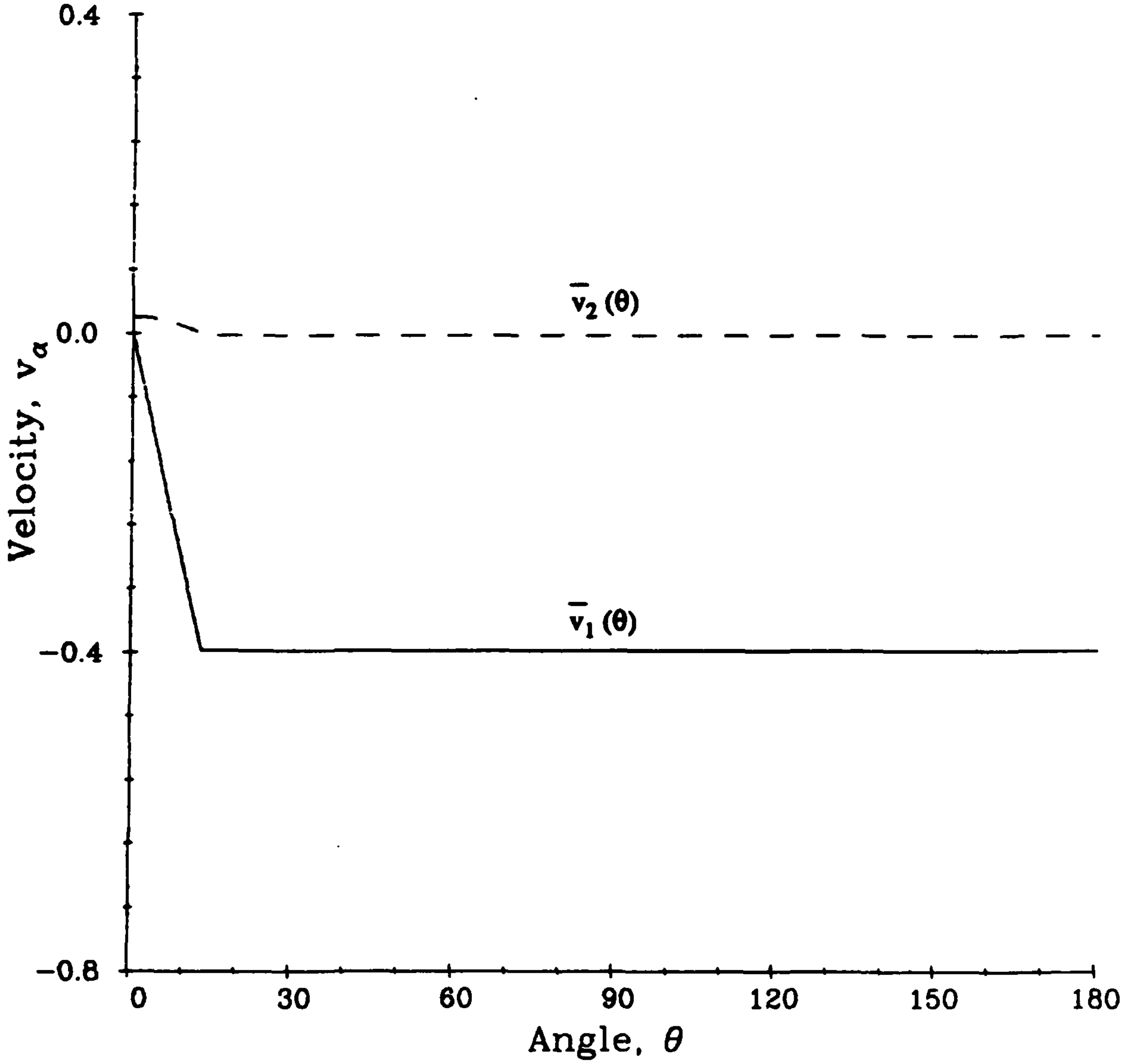

Figure 2. Velocity distribution

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